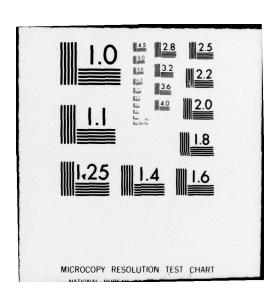
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# AFOSR-TR- 79-1157

#### Final Technical Report

Research Title: Effective Behavior of Composite Materials

Principal Investigator: George C. Papanicolaou

Period of Contract: August 1, 1978 to July 31, 1979

Grant Number: AFOSR-78-3668

#### Abstract

The following four problems have been analyzed in detail.

First, the effective heat removal rate for a conducting material perforated with many small holes is computed. Second, for a conducting material impregnated with nonconductors occupying a large fraction of the medium and randomly dispersed the effective conductivity is obtained. Third, for a material with a random rough boundary a theory of effective boundary impedance has been developed. Fourth, the effects of random structural imperfections on buckling loads of beams have been computed.

## Research objectives and statement of work

My research objectives and work are as follows:

- (i) Properties of boundaries and interfaces of composite materials.
- (ii) Wave propagation, heat conduction and other transport or dynamic effects in materials with amorphous or random structure and their effective macroscopic description.
- (iii) Linear and nonlinear (high intensity) electromagnetic effects in composites.

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AIR FORCE OFFICE OF SCIENTIFIC RESEARCH (AFSC)

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This technical report has been reviewed and is approved for public release IAW AFR 190-12 (7b). Distribution is unlimited.

A. D. BLOSE

Technical Information Officer

(iv) Effect of structural imperfections on the buckling loads of beams and related bifurcation problems.

During the period covered by the contract work was done in all four areas with most time spent in (ii) and (iv). Work in area (iii) is in progress. Work in area (i) was light and will be resumed shortly.

## 2. Status of research effort

The results of the research carried out can be described as follows:

(i) In [1] we set out to compute the effective heat removal rate for a conducting medium perforated with holes. The radius of the holes is small and the number of holes per unit volume is large so that their product is of order one. In this case the effect of the holes is measurable. Our anlysis involves rather delicate probabilistic estimates.

There are many other things we will be investigating in the area of perforated materials. One concerns the electromagnetic properties of such structures at low and high frequency. The other concerns strength properties (effective elastic constants).

(ii) In [2] we found the effective conductivity of a material impregnated with random inhomogeneities. What is new and different about our work is that first, the volume fraction occupied by inhomogeneities is not small (as it is in the case of holes) and

second, the conductivity changes significantly when passing from the conductor to the inhomogeneous material.

Work in this area will continue because the problem solved is only the simplest problem of the kind where random imperfections occur with order one volume fraction. We want to study electromagnetic, elastic and other properties. We also want to analyze fluctuations of the computed effective parameters.

(iii) In [3] we computed the effect of random imperfections on bifurcations. For a conductor subject to nonlinear joule heating by passage of current there is a critical value of the current beyond which equilibrium temperature distributions do not exist (breakdown). Suppose the conducting body has random imperfections which are not necessarily small but occur on a microscopic scale. How do they affect the onset of breakdown? In [3] we specifically answer this question by computing the statistics of the value of the current where breakdown may occur.

We intend to carry out many other computations based on the work in [3].

(iv) In [4] (for which a manuscript is in preparation) we have developed a theory for computing effective impedances for random rough surfaces. Our work differs from pervious work because we do not suppose that the average height of the rough surface is small or that the average curvature of the surface is large. We only assume that the correlation length of the surface is small. Our theory should be useful in finding proper-

ties of rough-cut outer surfaces of composites subject to acoustic or electromagnetic radiation. So far we have only considered the case of wavelengths large compared to the correlation length.

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#### 3. Written Publications

- [1] Diffusion in Regions with Many Small Holes, to be published by Springer Verlag (in the series of Lecture Notes in Mathematics) in the proceedings of the Vilnius Conference of 1978 (joint work with S. R. S. Varadhan).
- [2] Boundary value problems with rapidly oscillating random coefficients. To be published by North Holland in a book series edited by the Janos Bolyai Society carrying the proceedings of the annual (1979) conference on statistical mechanics (joint work with S. R. S. Varadhan).
- [3] Stochastically perturbed bifurcation. To be published by Springer Verlag for the IRIA Congress, Paris, France 1979.
- [4] Effective impedance for a random rough surface (in preparation).

## 4. Personnel associated with research effort

Michael Weinstein, a graduate student at the Courant Institute did some preliminary research on boundary value problems for symmetric hyperbolic systems during 1978-79.

He is expected to begin work on a thesis this fall.

Thesis topic will be in the area of perforated materials and their effective properties.

## 5. Interactions

During the contract period I was invited to lecture at several scientific meetings. Some of them are:

- (i) IEEE Systems Theory annual conference San Diego,
  January 1979, invited speaker (lectured on topics related to
  stochastic bifurcation).
- (ii) Rome, Italy, department of Mathematics, June 1979; lectured on all the problems stated.
  - (iii) Budapest, Hungary, June 1979, lectured on [2].
  - (iv) Paris, France (Dec. 1979). Will lecture on [3].